MATH 245 F20, Exam 2 Questions
(60 minutes, open book, open notes)

1. Freebie.
2. Prove that $\forall n \in \mathbb{Z}$, we must have $\frac{(n+1)(n-2)}{2} \in \mathbb{Z}$.
3. Let $x \in \mathbb{R}$. Prove that TFAE: (a) x is rational; (b) $7x$ is rational; (c) $x+1$ is rational.
4. Prove or disprove: $\forall x \in \mathbb{R}, \lfloor x \rfloor = -\lceil -x \rceil$.
5. Prove that $\forall n \in \mathbb{N}, 9^n > n^3$.
6. Prove that, for every $n \in \mathbb{N}$, the Fibonacci numbers satisfy $F_{n+3} = 2 + \sum_{i=2}^{n+1} F_i$.

   Pick your favorite, different, real numbers b, c that are not integers, to use in the rest of the exam.

7. Using your favorite b, c: solve the recurrence with initial conditions $a_0 = b, a_1 = c$ and relation $a_n = 2a_{n-1} - a_{n-2}$ (for $n \geq 2$).
8. Using your favorite b, c: (i) Prove or disprove that $n^b = O(n^c)$; and (ii) Prove or disprove that $n^c = O(n^b)$.